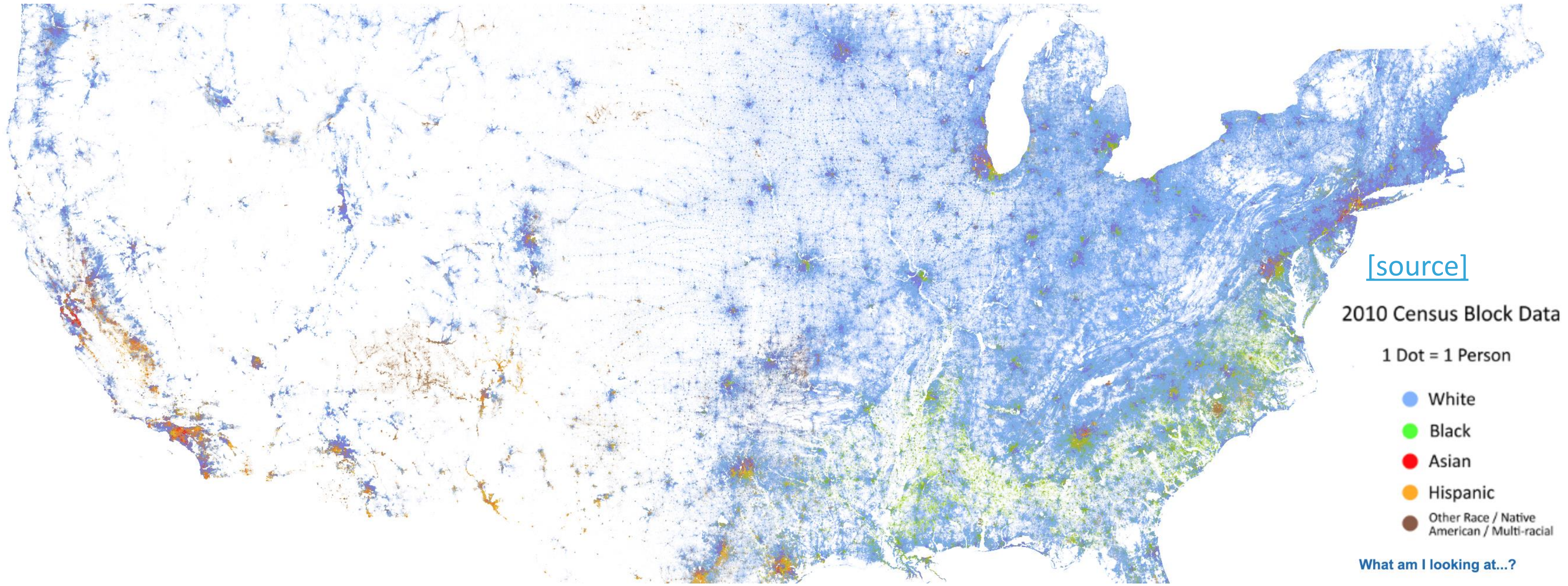
# *Spatial* Data Science

## Exploring Space in Data

Lecture 4




Theodoros Chatzivasileiadis

The Racial Dot Map developed by the Demographics Research Group at University of Virginia

# *Space, formally*

For a statistical method to be **explicitly spatial**, it needs to contain some representation of the geography, or **spatial context**

One of the most common ways is through **Spatial Weights Matrices**

- **(Geo)Visualization**: translating numbers into a (visual) language that the human brain "*speaks better*"
- **Spatial Weights Matrices**: translating geography into a (numerical) language that a computer "*speaks better*".

Core element in several spatial analysis techniques:

- Spatial autocorrelation
- Spatial clustering / geodemographics
- Spatial regression

# *W* as a formal representation of Space

# $W$

*N x N positive matrix that contains spatial relations between all the observations in the sample*

$$w_{ij} = \begin{cases} x > 0, & if\ i\ and\ j\ are\ neighbours \\ 0, & otherwise \end{cases}$$

$w_{ii}$ *= 0 by convention*

*…What is a neighbour???*

* kindly do not lie with maps

# Types of $W$

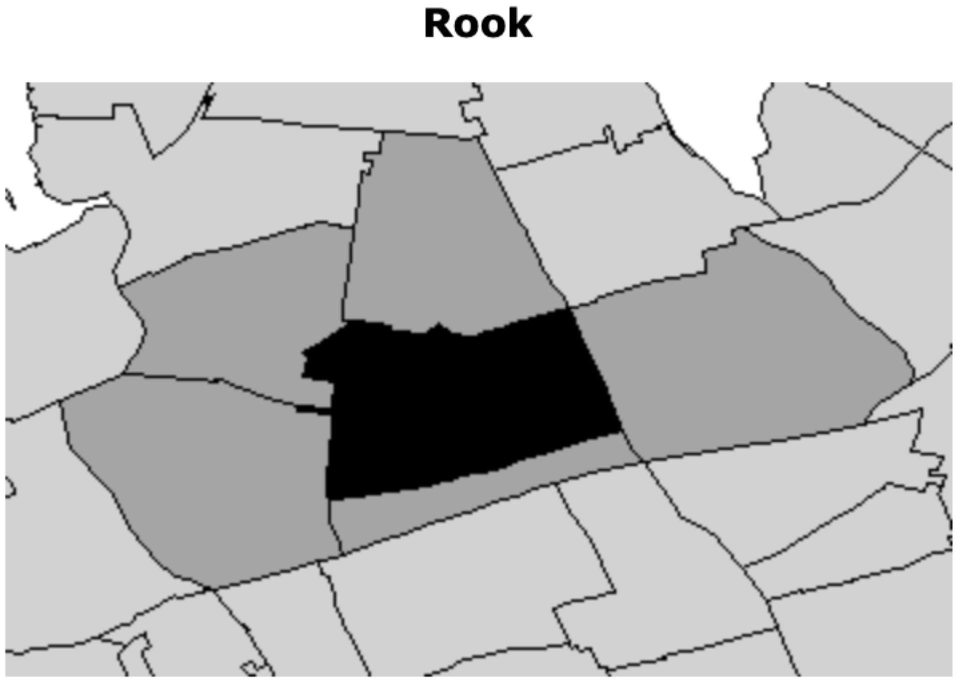
A neighbour is “somebody” who is

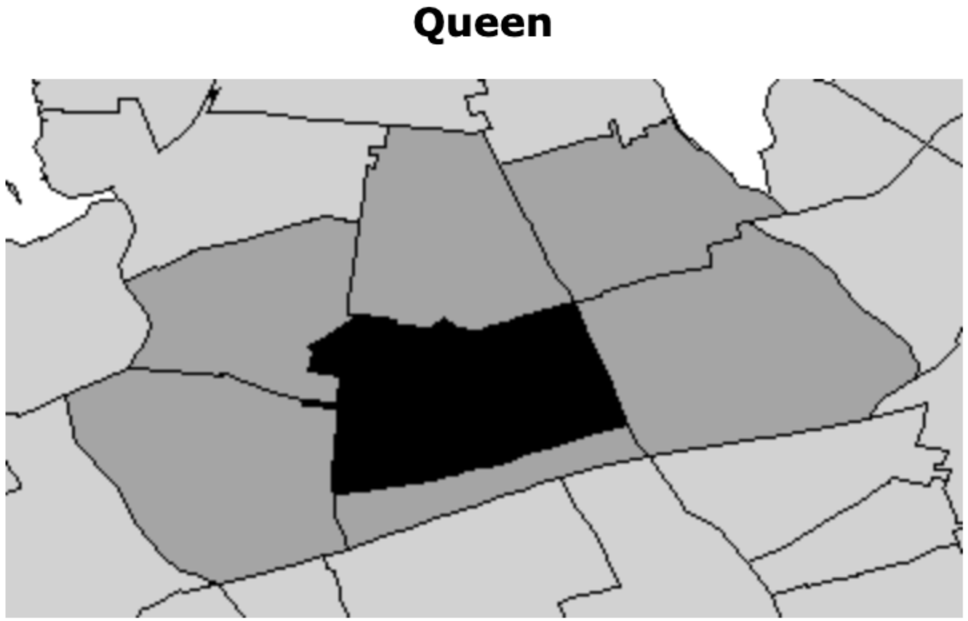
- Next door → **Contiguity**-based Ws
- Close → **Distance**-based Ws
- In the same “place ” as us → **Block** weights

# Contiguity-based weights

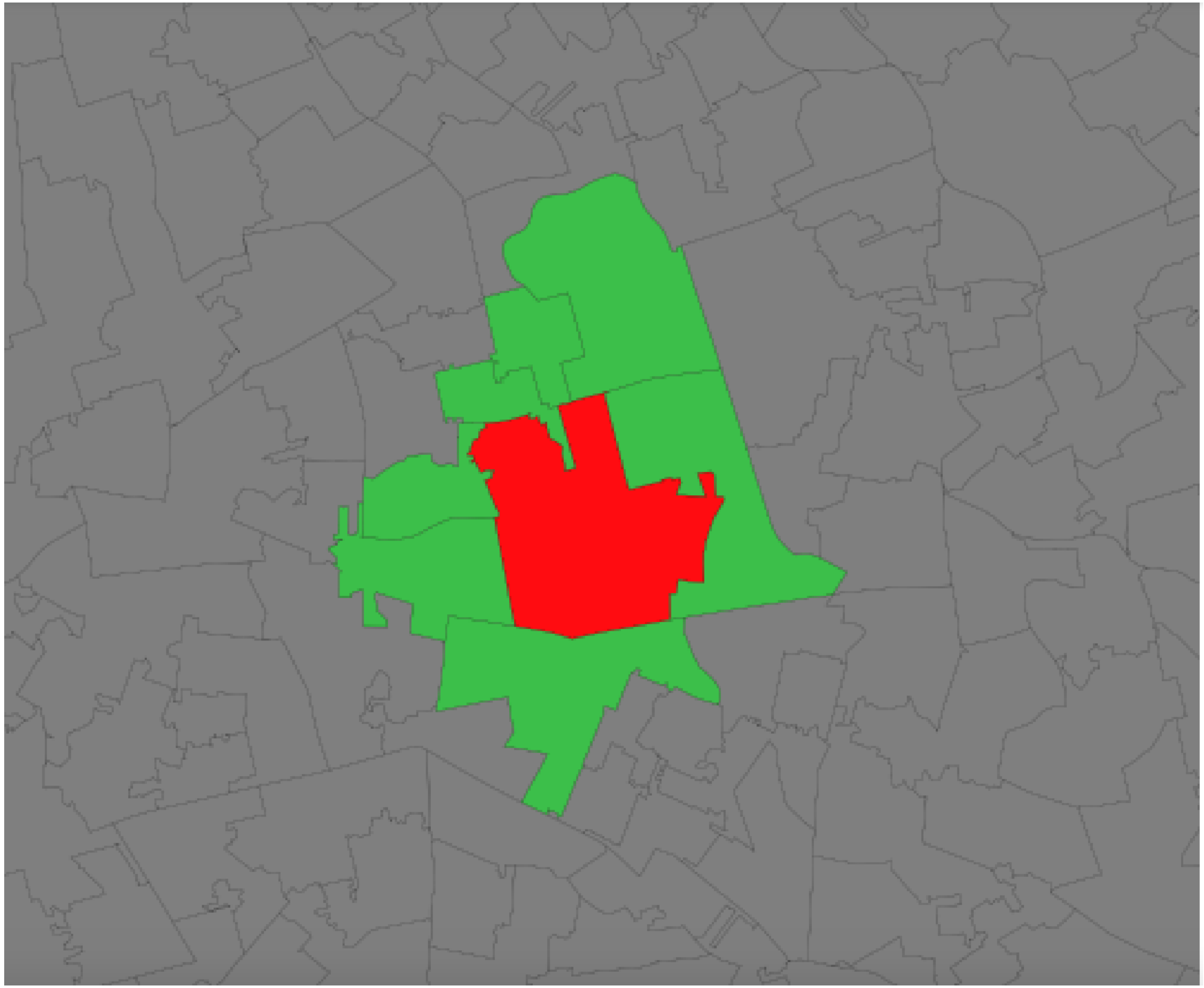
Sharing **boundaries** to any extent

- Rook
- Queen
- …




Images taken from: Arribas-Bel, D. (2019). A course on geographic data science. *Journal of Open Source Education*, *2*(16), 42.



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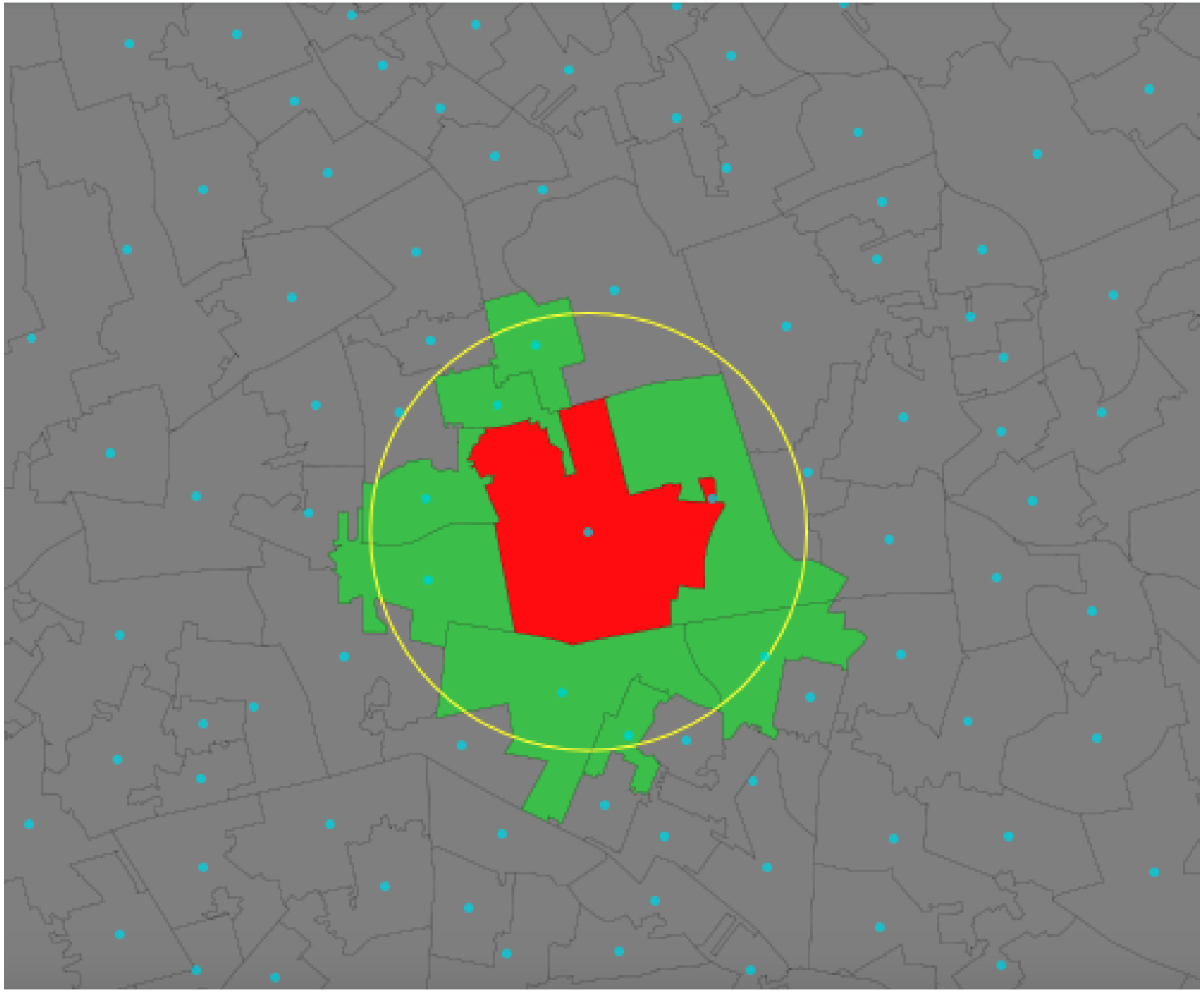
# Distance-based weights

Weight is (inversely) proportional to distance between observations

- Inverse distance (threshold)



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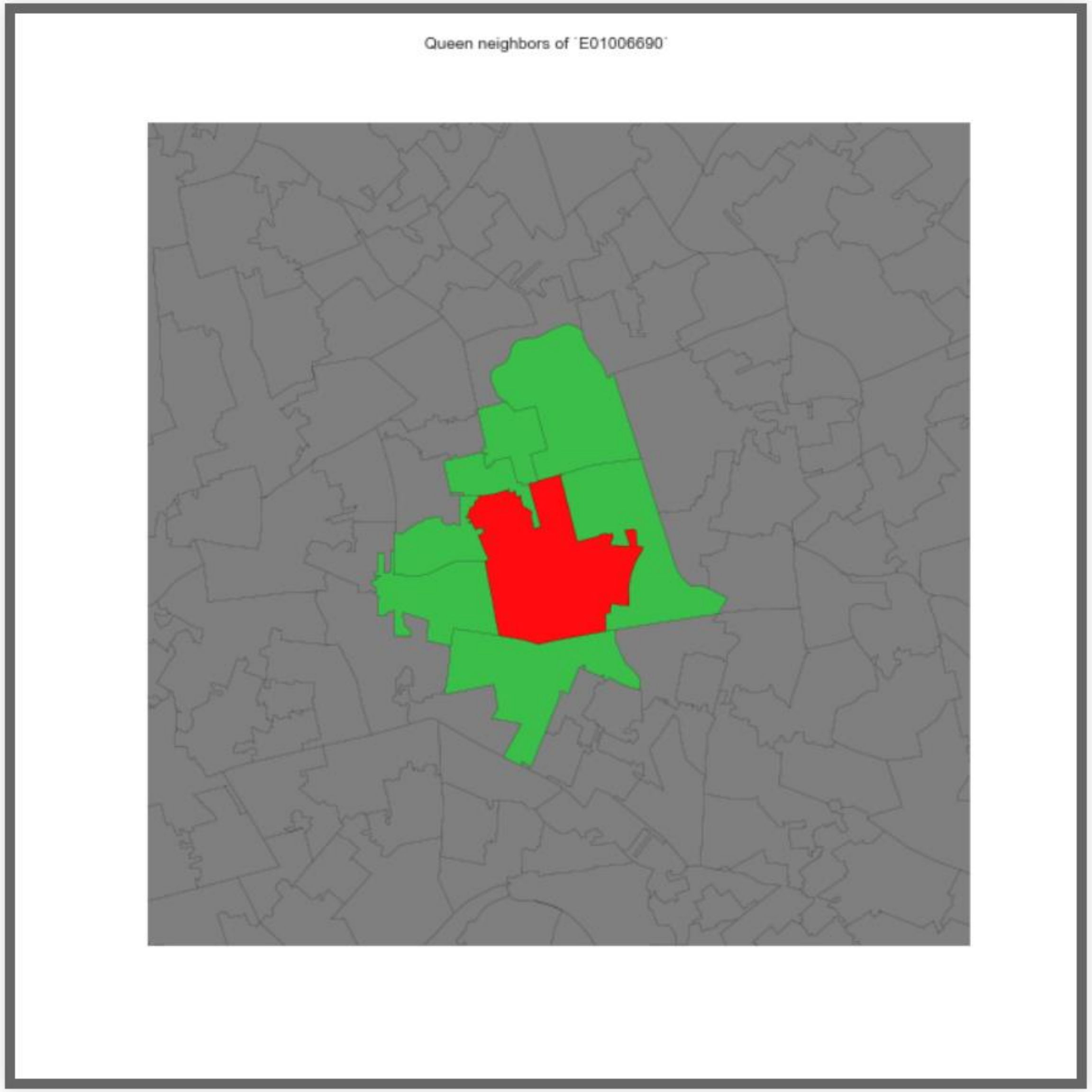
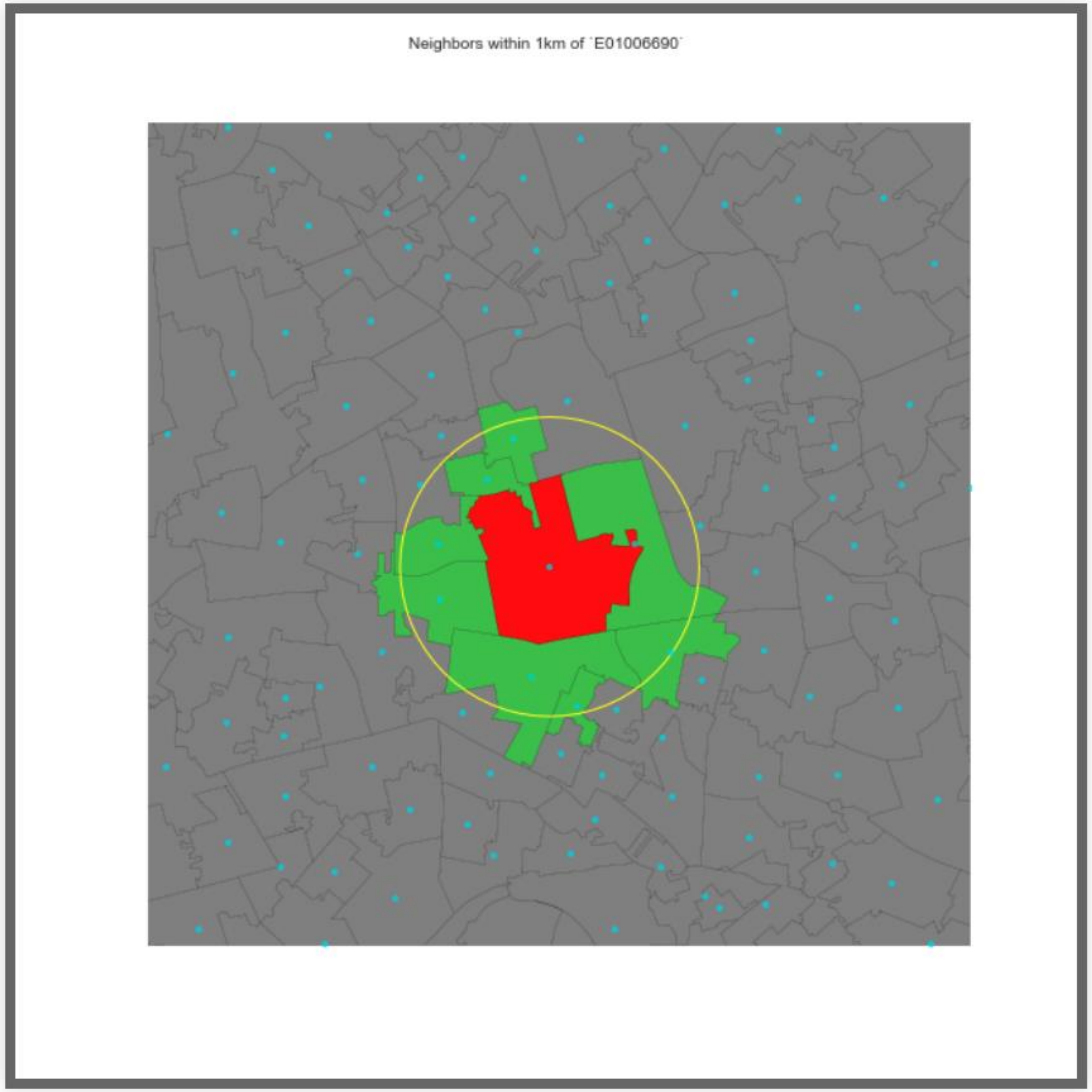



Images taken from: Arribas-Bel, D. (2019). A course on geographic data science. *Journal of Open Source Education*, *2*(16), 42.

# Block weights

Weights are assigned based on discretionary rules loosely related to geography

For example:

- Buurts into Wijks
- Post-codes within city boundaries
- Counties within states
- …

# How much of a neighbour?

**Not a neighbour?** receive zero weight: $w_{ij}$ = 0

Neighbours, it depends, $w_{ij}$ can be:

- One: $w_{ij}$ = 1 → Binary
- Some proportion (0 < $w_{ij}$ < 1, continuous) which can be a function of:
  - Distance
  - Strength of interaction (e.g., commuting flows, trade, etc.)

# Choice of $W$

Should be based on and reflect the **underlying channels of interaction** for the question at hand.

Examples:

- Processes propagated by immediate contact (e.g. disease contagion) → Contiguity weights
- Accessibility → Distance weights
- Effects of county differences in laws → Block weights

# Standardisation

In some applications (e.g. spatial autocorrelation) it is common to *standardize* W

The most widely used standardization is row-based: divide every element by the sum of the row:

$$w'_{ij} = \frac{w_{ij}}{w_{i.}}$$

where $w_{i.}$ is the sum of a row

# Spatial Lag

# Spatial Lag

Weighted average of neighbouring values

- Neighbour definition comes from spatial weights $w_{ij}$

$$Y_{iL} = w_{i1}Y_1 + w_{i2}Y_2 + w_{i3}Y_3 + \ldots\ w_{in}Y_n$$

Spatial Lag variable has a *smaller* variance than Y because it is a smoother function

# Spatial Lag

- Measure that captures the behaviour of a variable in the neighborhood of a given observation i.
- If W is standardized, the spatial lag is the weighted average value of the variable in the neighborhood (good for comparison and scaling)

# Spatial Lag

- Common way to introduce space formally in a statistical framework
- Heavily used in both **ESDA** and spatial regression to delineate neighborhoods.
- Examples (covered in next lecture):
  - Moran’s I
  - LISAs
  - Spatial models (lag, error…)

# Recapitulation

- Everything is connected and must be considered so
- Spatial Weights matrices: matrix encapsulation of space
- Different types for different cases (contiguous, distance and blocks)
- Useful in many contexts, like the spatial lag and Moran plot, but also many other things!

# Today

- Exploratory Spatial Data Analysis (ESDA)
- Spatial Autocorrelation Measures
  - Global
  - Local

# [Exploratory]

Focus on discovery and assumption-free investigation

# [Spatial]

Patterns and processes that put space and geography at the core

# [Data Analysis]

Statistical techniques

Patel, R., Verma, T., Marvuglia, A., Huang, Y., Baustert, P., Shivakumar, A., Nikolic, I. (2021). Quantifying the Consumption-driven Environmental Impact of Households in Cities. In Preparation (In Preparation).

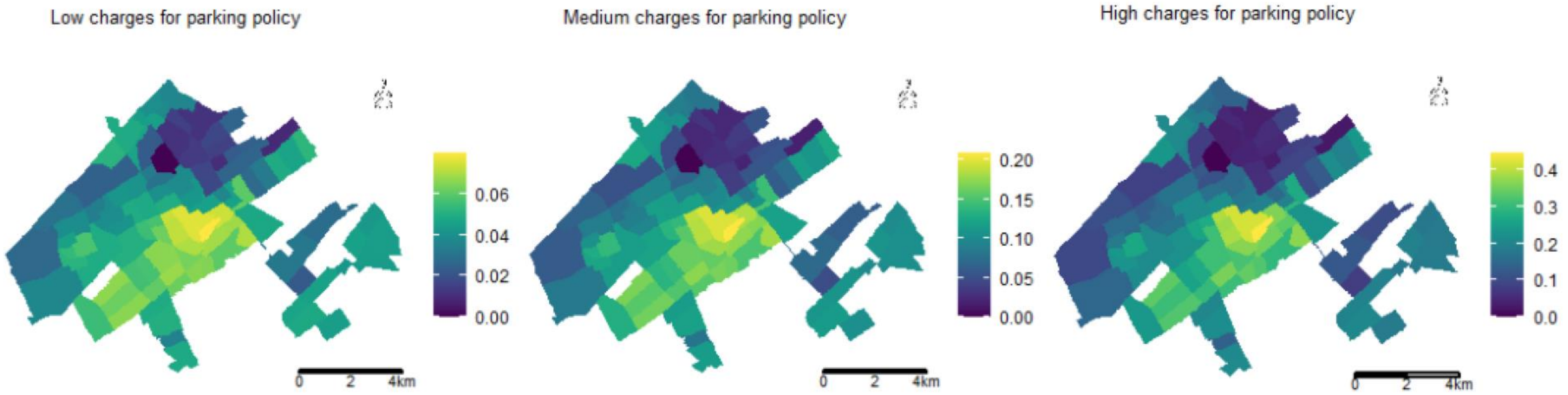
# Questions that ESDA helps with...

## Answer

- Is the variable I'm looking at concentrated over space?
- Do similar values tend to locate close by?
- Can I identify any particular areas where certain values are clustered?

## Ask

- What is behind this pattern?
- What could be generating the process?
- Why do we observe certain clusters over space?



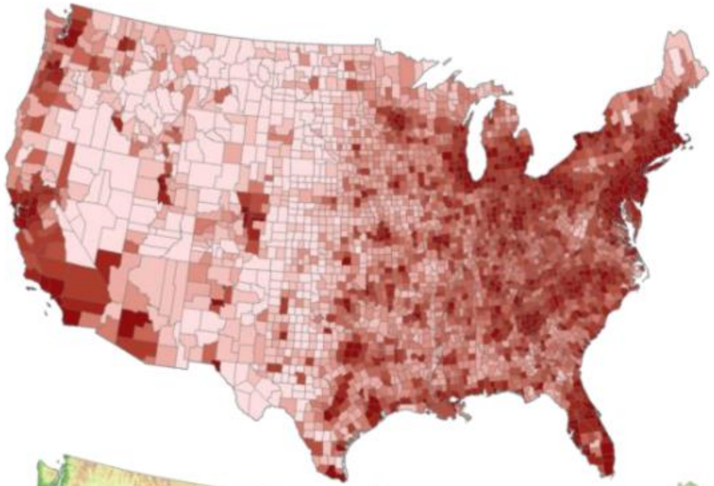

*Net emission reduction in the mobility sector for different neighbourhoods of the Hague under different car parking charging policies ceteris paribus*

The first law of geography:

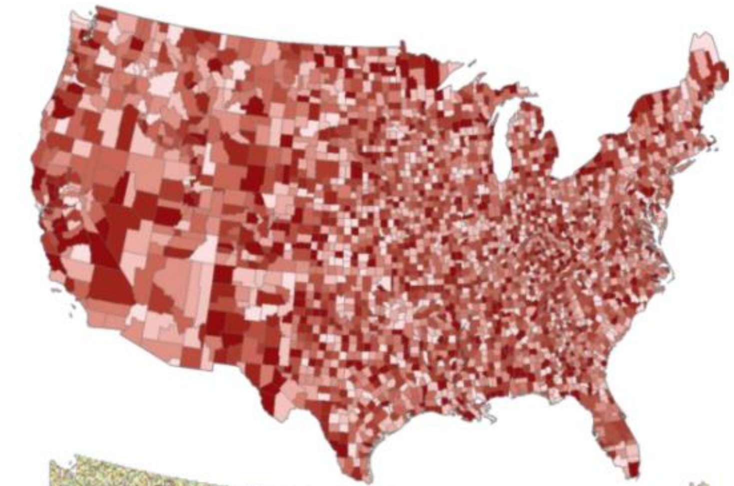
*“Everything is related to everything else, but near things are more related than distant things.”*

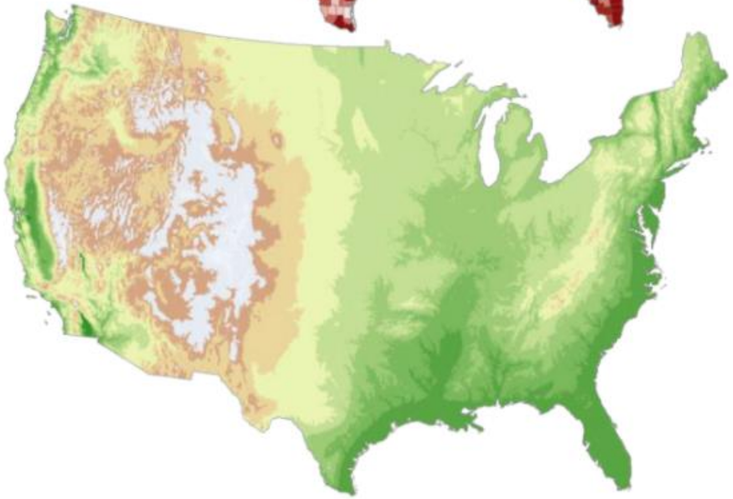
Waldo R. Tobler (Tobler 1970)

If features were randomly distributed



population density map of the US

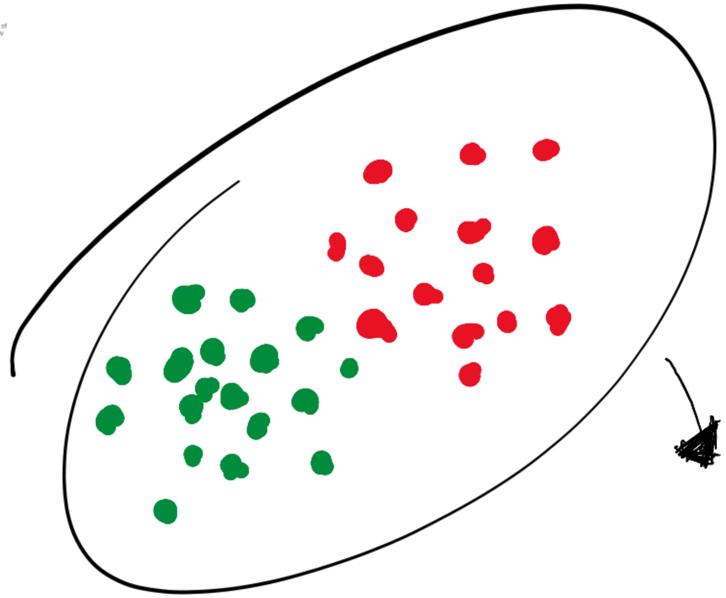




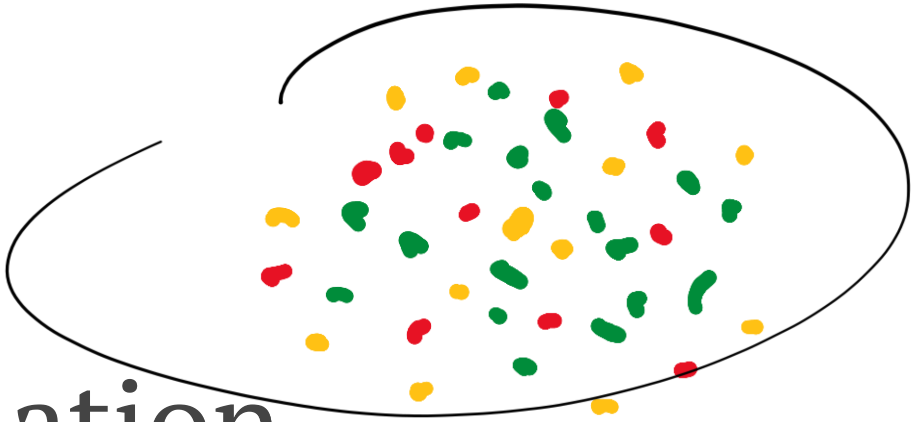
elevation map of the US

HOW ARE FEATURES CLUSTERED?

Images adapted from: Gimond, Manuel. Intro to GIS and Spatial Analysis.



Clustered



non-clustered regions

# Spatial Autocorrelation

1. Quantitative
2. Objective
3. Degree of similarity
4. Where does it occur?

# Spatial Autocorrelation

- Statistical representation of Tobler’s law
- Spatial counterpart of traditional correlation

***Degree to which similar values are located in similar locations***

# Spatial Autocorrelation

Two flavours:

- Positive: similar values → similar location *(close by)*
- Negative: similar values → dissimilar location *(further apart)*

# Examples

**Positive** SA: income, poverty, vegetation, temperature…

**Negative** SA: supermarkets, police stations, fire stations, hospitals…

# Scales

[Global] Clustering: do values tend to be close to other (dis)similar values?

[Local] Clusters: are there any specific parts of a map with an extraordinary concentration of (dis)similar values?

# Global Spatial Autocorrelation

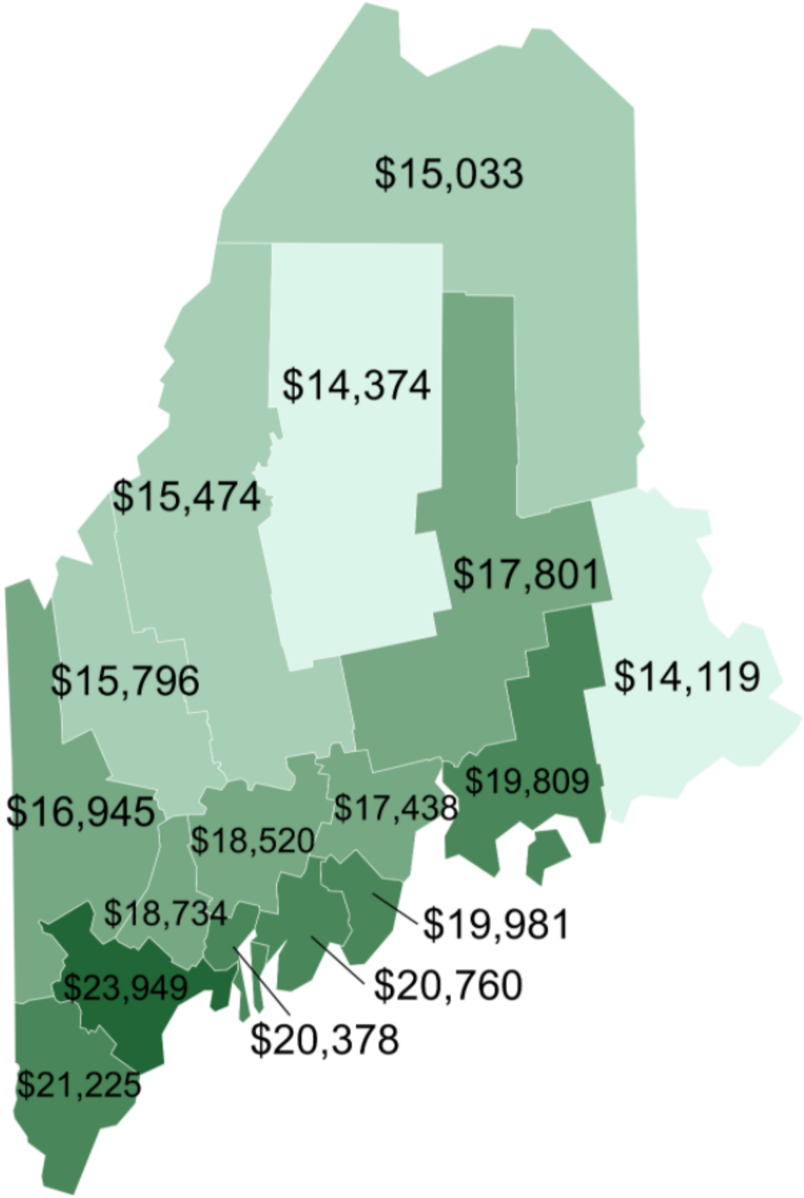
# Global Spatial Autocorr.

"Clustering"
*Overall trend where the distribution of values follows a particular pattern over space*

**[Positive]** Similar values close to each other (high-high, low-low)
**[Negative]** Similar values far from each other (high-low)

How to measure it???

Let's start with a working example: 2010 per capita income for the state of Maine.

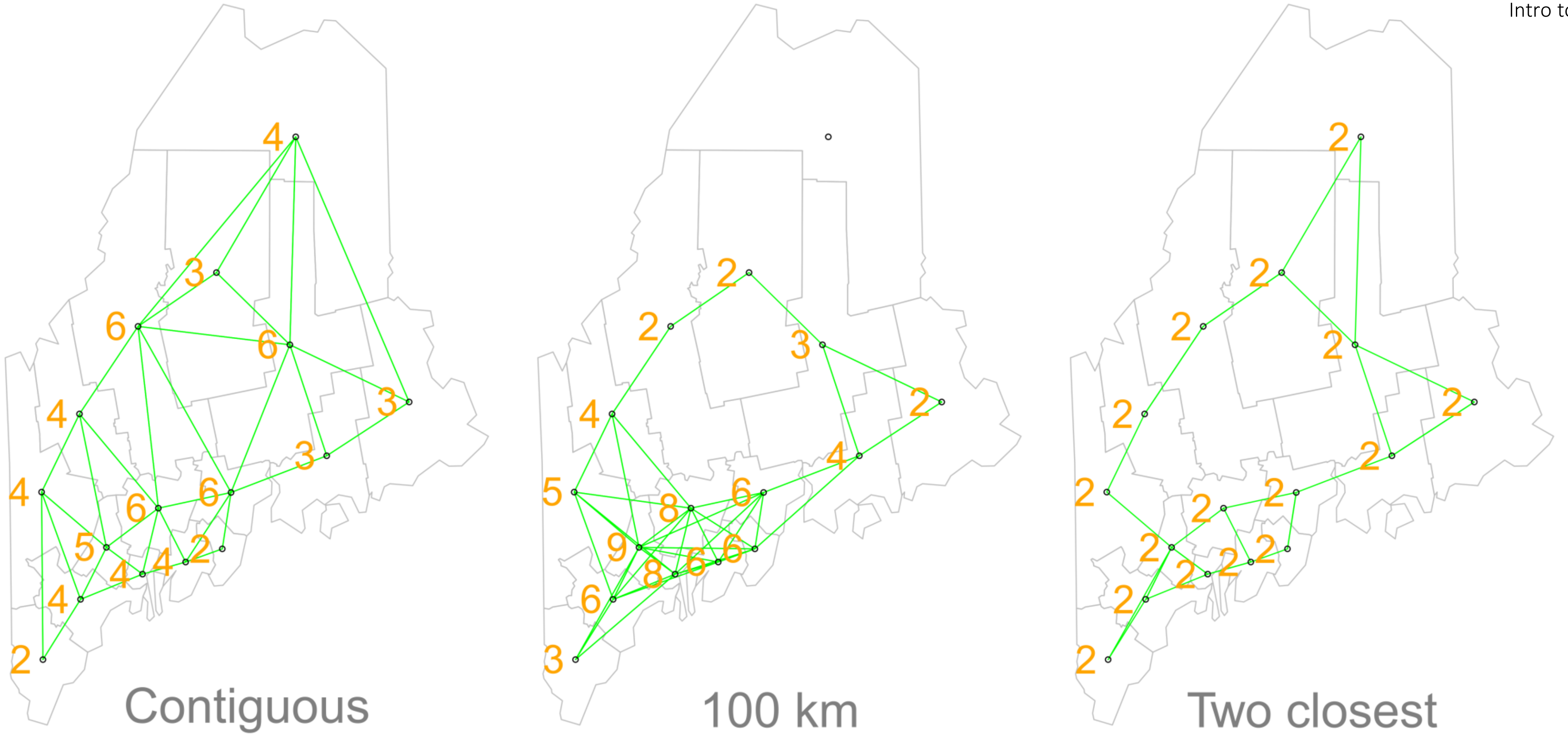



Images adapted from: Gimond, Manuel.
Intro to GIS and Spatial Analysis.

# Moran Plot

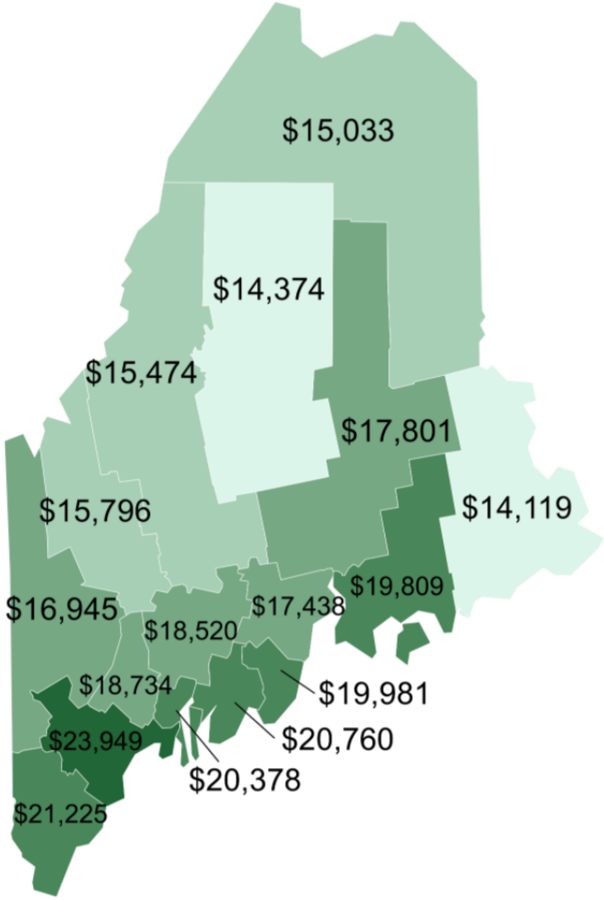
- Graphical device that displays a **variable** on the horizontal axis against **its spatial lag (Y_il – previous lecture)** on the vertical one
- Variable and spatial weights matrix are preferably standardized
- Assessment of the overall association between a variable in each location and, in its *neighbourhood*

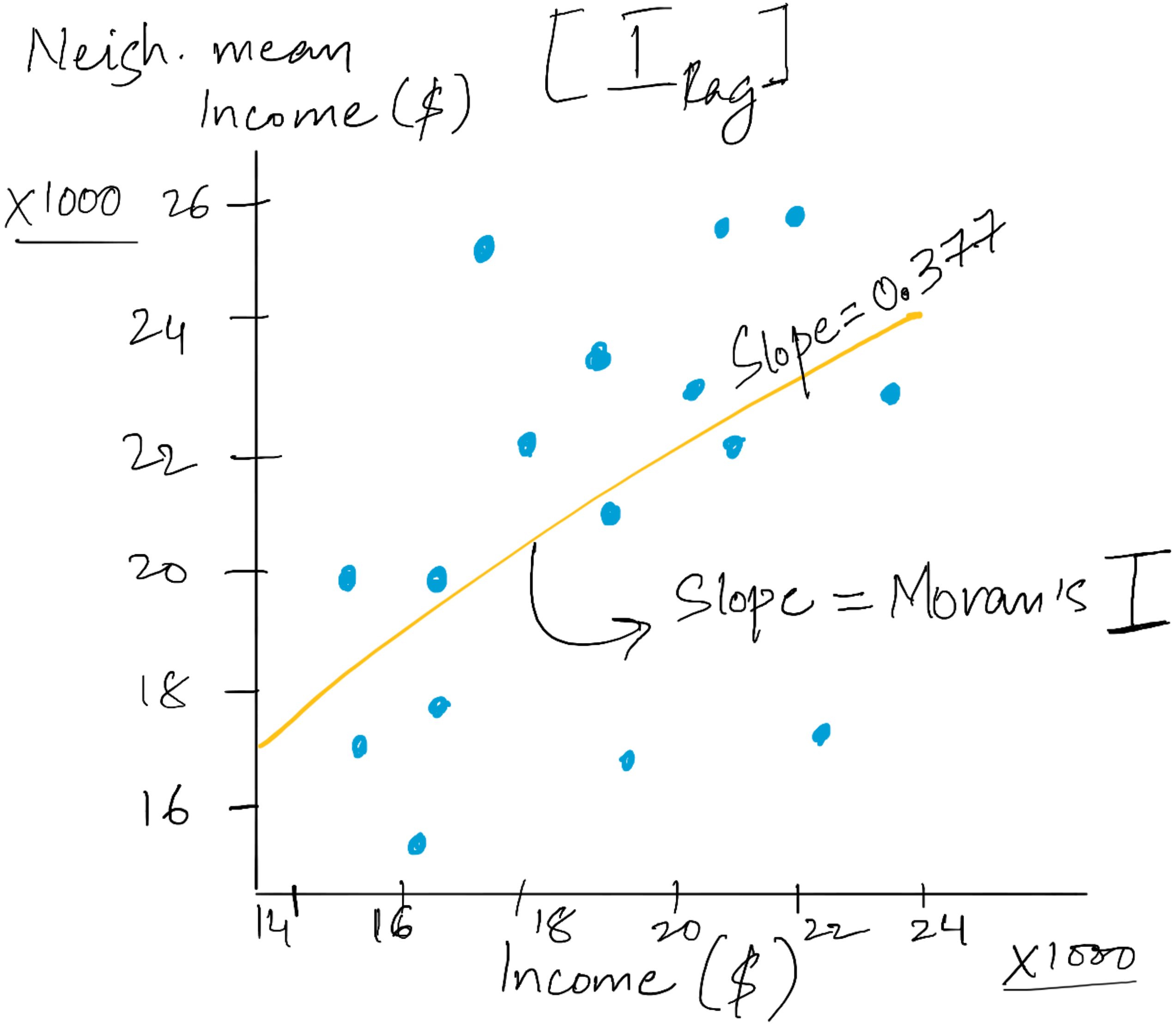
Images adapted from: Gimond, Manuel.
Intro to GIS and Spatial Analysis.




Maps show the links between each polygon and their respective neighbour(s) based on the neighbourhood definition. A contiguous neighbour is defined as one that shares a boundary or a vertex with the polygon of interest. Orange numbers indicate the number of neighbours for each polygon. Note that the top most county has no neighbours when a neighbourhood definition of a 100 km distance band is used (i.e. no centroids are within a 100 km search radius)

Let's start with a working example: 2010 per capita income for the state of Maine.

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Intro to GIS and Spatial Analysis.

# Moran's I

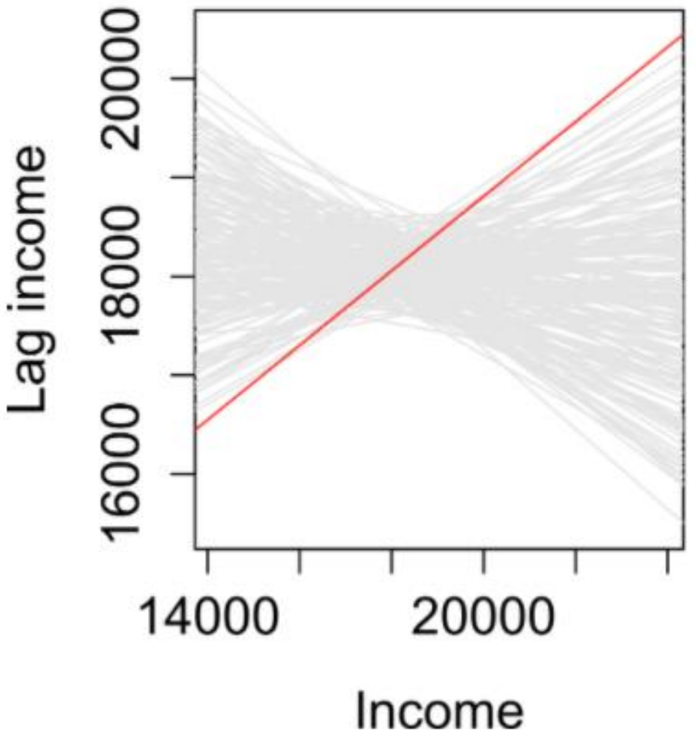
- Formal test of global spatial autocorrelation
- Statistically identify the presence of clustering in a variable
- Slope of the Moran plot
- Inference based on how likely it is to obtain a map like the observed one from a purely random pattern

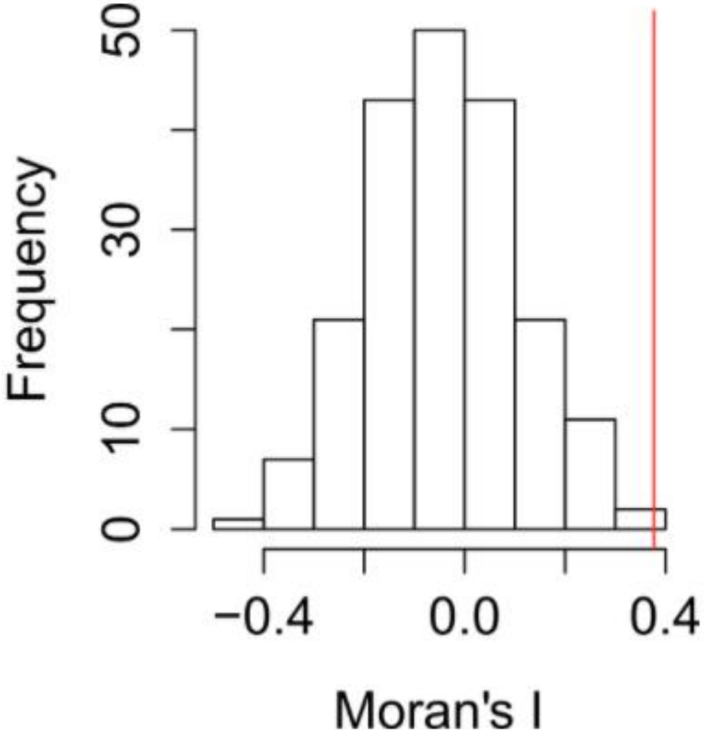
$$I = \frac{\sum_i \sum_j w_{ij} z_i . z_j}{\sum_i z_i^2} = \frac{\sum_i (z_i \times \sum_j w_{ij} z_j)}{\sum_i z_i^2}.$$

$I \propto$ Assumptions in $W$

# How significant is this I statistic?

- Permutation method – Monte Carlo
- Null hypothesis $H_{0:}$
  Attribute is randomly distributed

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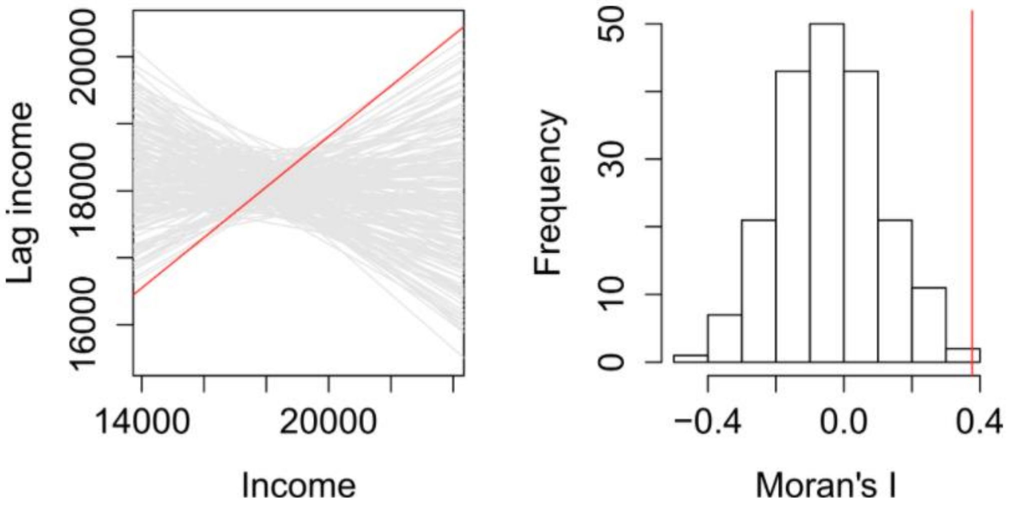
# How significant is this I statistic?

Pseudo p-value $$\frac{N_{extreme} + 1}{N + 1}$$

- where $N_{extreme}$ is the number of simulated Moran's I values more extreme than our observation
- N is the total number of simulations.
- Here, out of 199 simulations,
- $N_{extreme}$ = 1, so p is equal to (1 + 1) / (199 + 1) = 0.01.
- This is interpreted as *"there is a 1% probability that we would be wrong in rejecting the null hypothesis $H_o$."*

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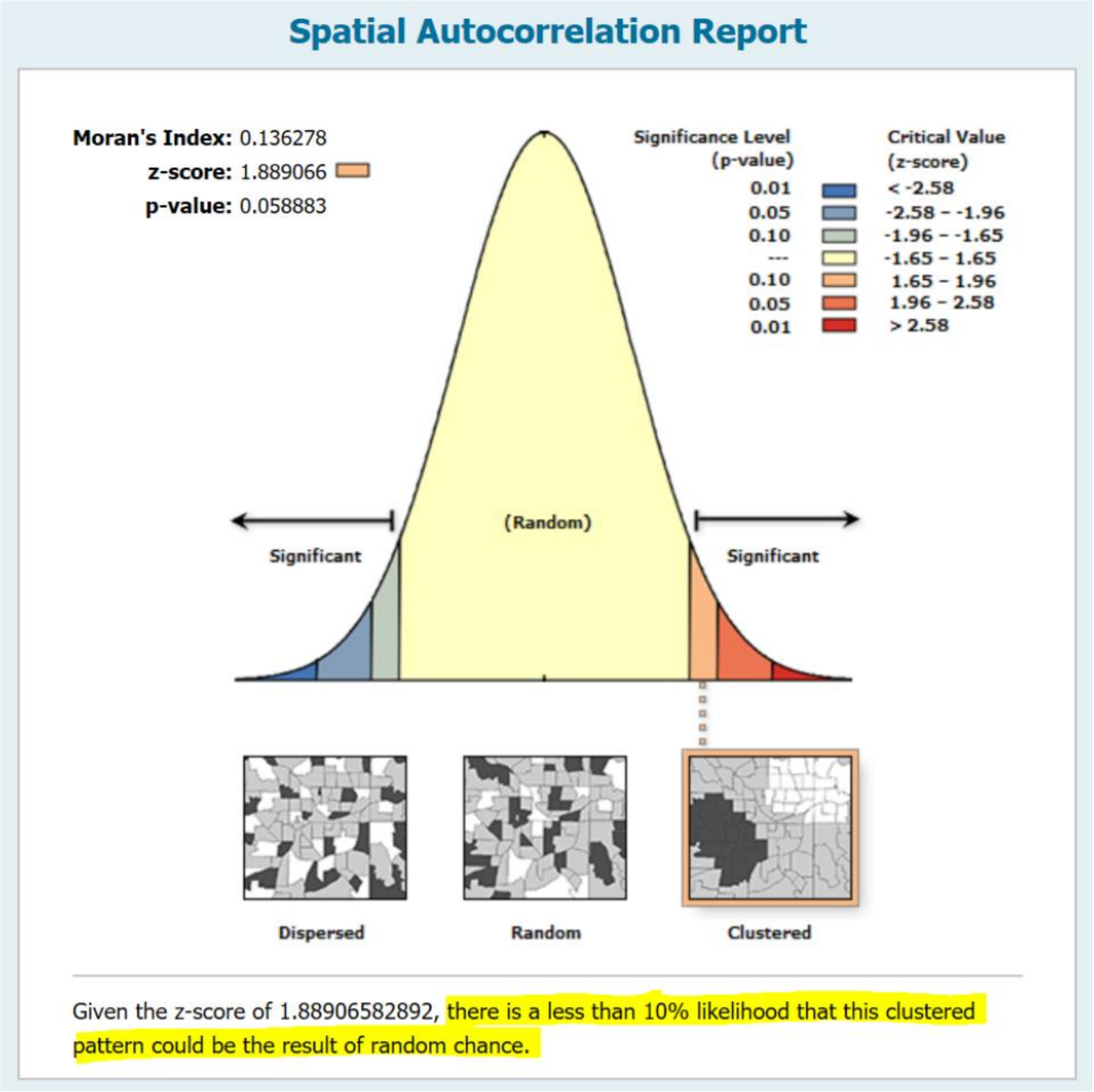
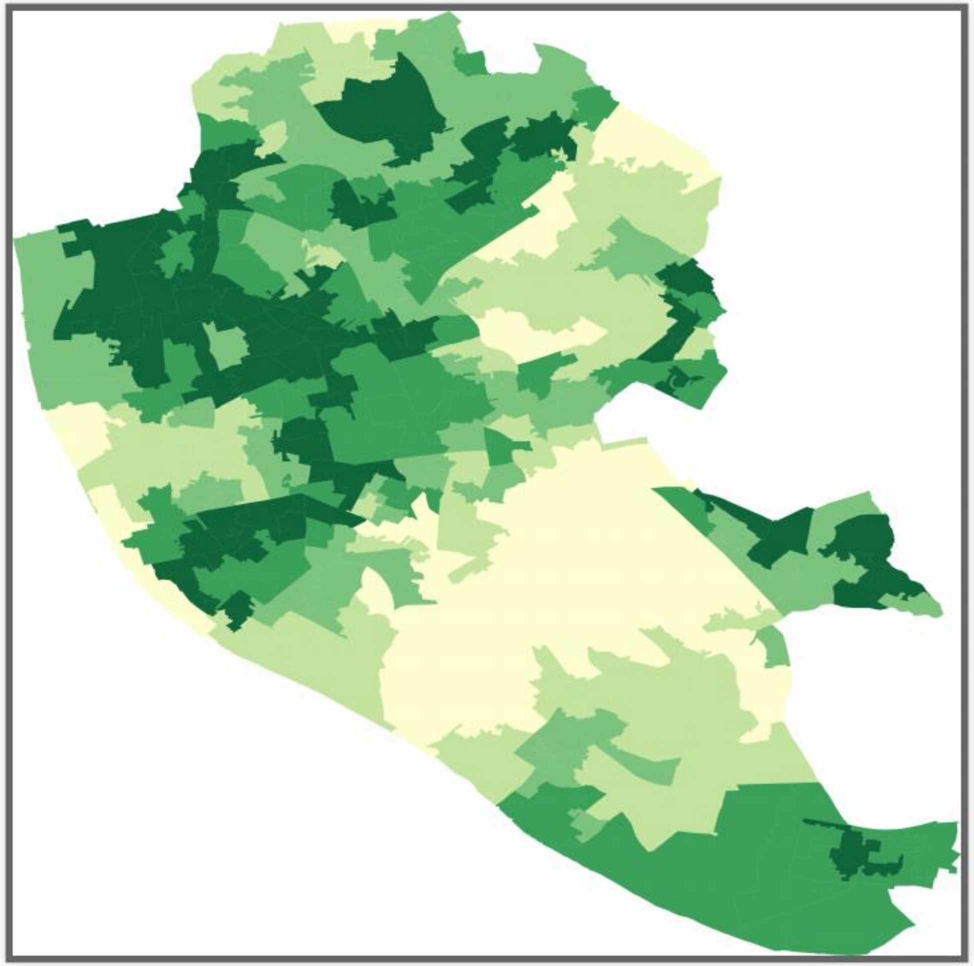
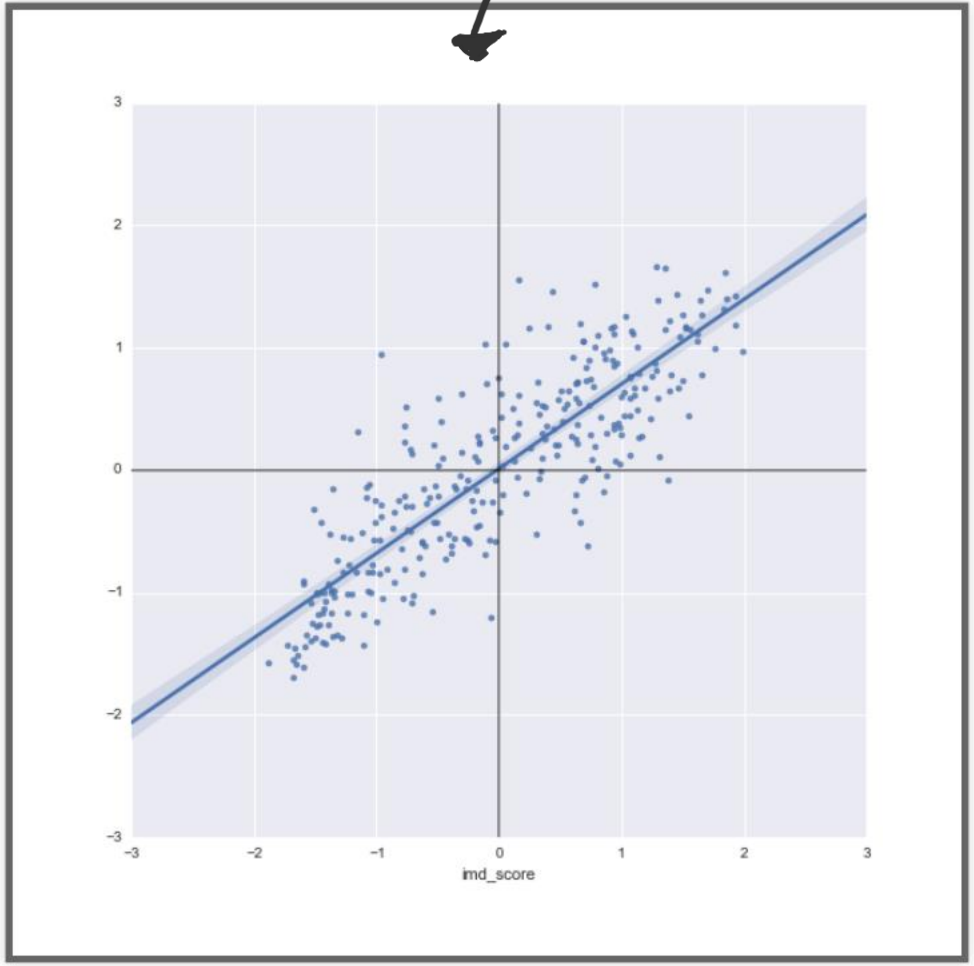
How do we understand the statistic?



from the lab exercises



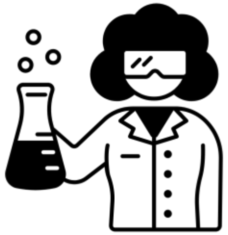



Images taken from: Arribas-Bel, D. (2019). A course on geographic data science. *Journal of Open Source Education*, *2*(16), 42.

# Break

CHILL

WALK



COFFEE OR TEA

MAKE FRIENDS

Icons by Iconathon on The Noun Project

# Local Spatial Autocorrelation

# Local Spatial Autocorr.

"Clusters"

*Pockets of spatial instability*

Portions of a map where values are correlated in a particularly strong and specific way

**[High-High]** + SA of high values (hotspots)

**[Low-Low]** + SA of low values (coldspots)

**[High-Low]** - SA (spatial outliers)

**[Low-High]** - SA (spatial outliers)

# What is LISA?

**L**ocal **I**ndicators of **S**patial **A**ssociation

- Statistical tests for ***spatial cluster detection*** → Statistical significance
- **Compares** the **observed** map with many **randomly** generated ones to see how likely it is to obtain the areas of unusually high concentration

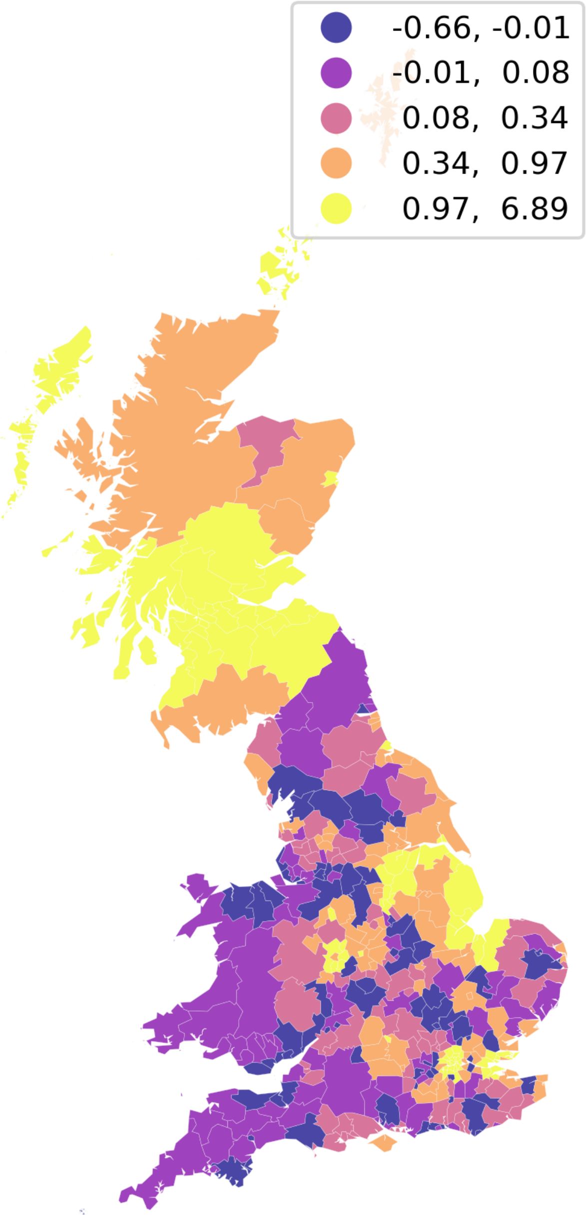
# What is LISA?

$$I = \frac{\sum_i \sum_j w_{ij} z_i . z_j}{\sum_i z_i^2} = \frac{\sum_i (z_i \times \sum_j w_{ij} z_j)}{\sum_i z_i^2}.$$

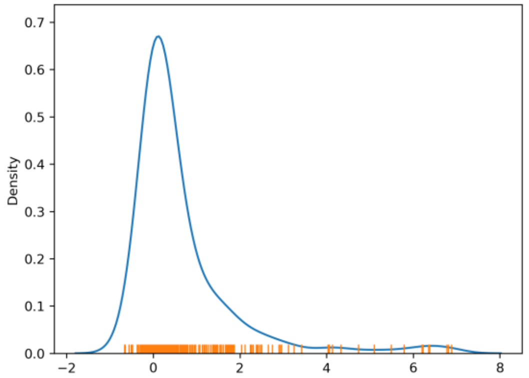
$$I_i = c . z_i \sum_j w_{ij} z_j,$$

The values in the **left tail** of the density represent locations **displaying negative spatial association**. There are also two forms, a **high value surrounded by low values**, or a **low value surrounded by high-valued** neighboring observations. And, again, the statistic cannot distinguish between the two cases.

# HL/LH



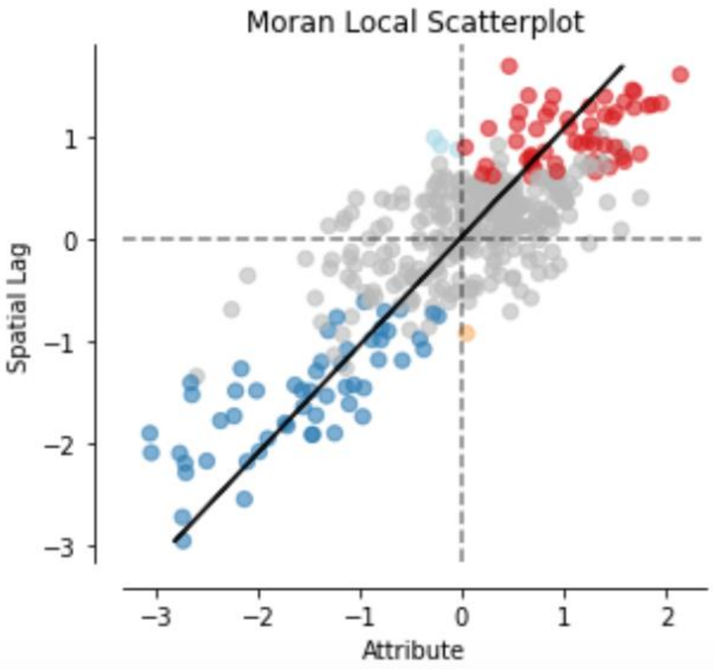

Local Statistics

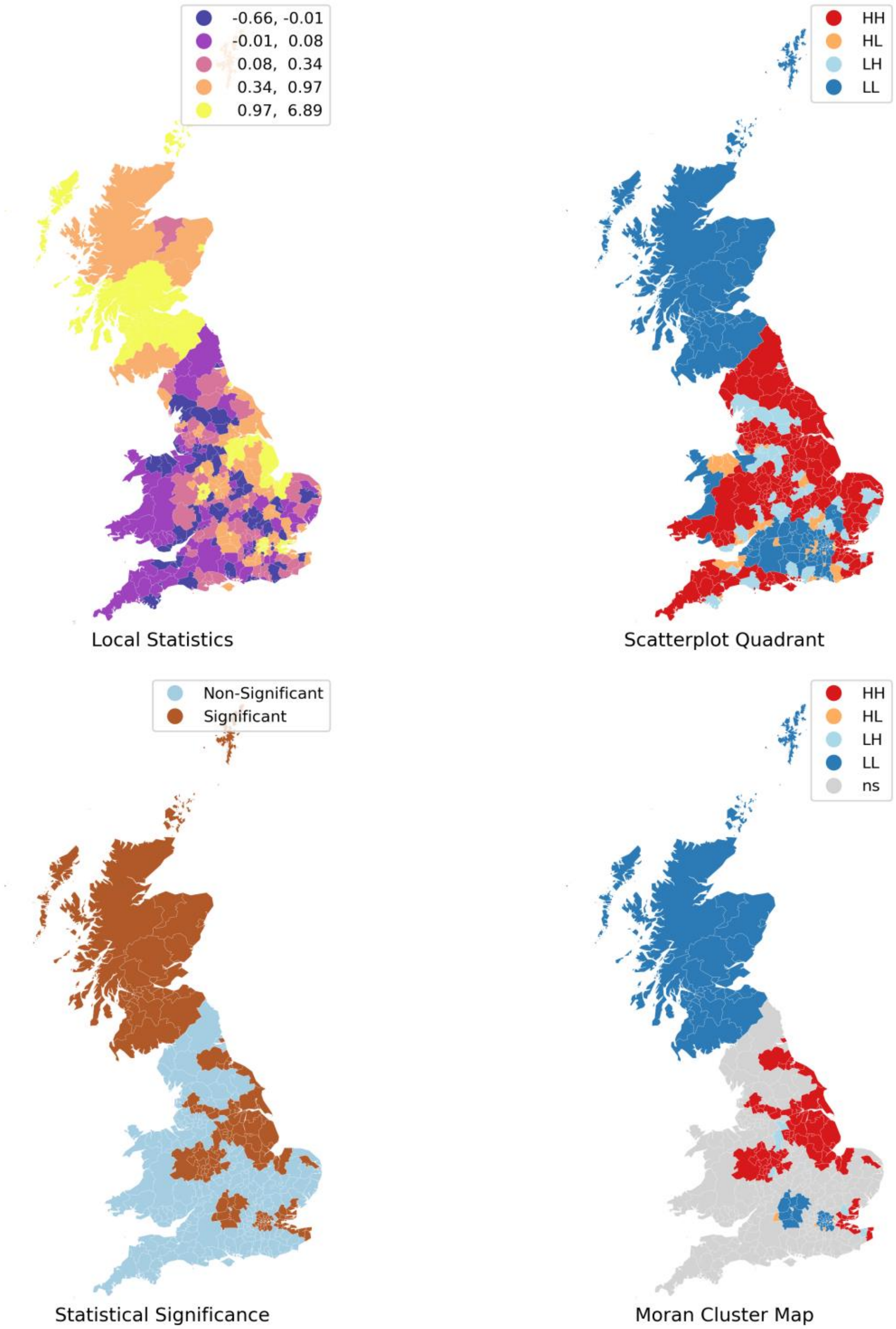



Here it is important to keep in mind that the **high positive values** arise from **value similarity** in space, and this can be due to either **high values being next to high values** *or* **low values next to low values**. The local values alone cannot distinguish these two cases.

# HH/LL

Images taken from: Arribas-Bel, D. (2019). A course on geographic data science. *Journal of Open Source Education*, *2*(16), 42.


Moran Local Scatterplot
Spatial Lag
Attribute
1
0
−1
−2
−3
−3
−2
−1
0
1
2


-0.66, -0.01
-0.01, 0.08
0.08, 0.34
0.34, 0.97
0.97, 6.89
Local Statistics
HH
HL
LH
LL
Scatterplot Quadrant
Non-Significant
Significant
Statistical Significance
HH
HL
LH
LL
ns
Moran Cluster Map

# Recapitulation

**ESDA** is a family of techniques to explore and spatially interrogate data

Main function: characterise **spatial autocorrelation**, which can be explored:

- **Globally** (e.g. Moran Plot, Moran ’ s I)
- **Locally** (e.g. LISAs)

# For next class..

**Finish** Labs to practice programming

**Complete** Homework for more practice

**Check** Assignment contents and due date

**See** "To do before class" for next lecture (~ 1 hour of self-study)